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Substituting this value of RP/FP in (4), we have

$$\frac{DP}{FP}:\frac{AP}{CP}=\frac{DE}{EF}:K,$$

or

$$\frac{DP}{FP}:\frac{DF}{EF}=\frac{AP}{CP}:K. \qquad (5)$$

But the anharmonic ratio of all transversals cutting any given pencil of four lines, as $O-A'B'C'P$, is a constant. Hence,

$$\frac{DP}{FP}:\frac{DE}{EF}=\frac{AP}{CP}:\frac{AB}{BC}. \qquad (6)$$

From (5) and (6), $AB/BC = K$. Hence AP is the transversal required.

Also solved by the PROPOSER.

453. Proposed by CLIFFORD N. MILLS, Brookings, S. D.

Prove geometrically the formulæ for $\sin 2\beta$, $\cos 2\beta$, $\sin 3\beta$, $\cos 3\beta$.

SOLUTION BY THE PROPOSER.

(1) $\sin 2\beta = 2\sin\beta\cos\beta$.

Inscribe in a circle any triangle, ABC, with the angle at B equal to 2β (Fig. 1). Draw AM through the center of the circle; BK, the bisector of the angle B; OK, the radius of the circle; AK, KM, and KC, K being on the circumference of the circle.

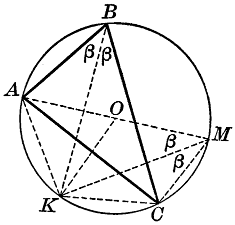

Fig. 1.

Then

$$\sin 2\beta = \frac{AC}{2r}.$$

$\triangle$s AKC and KOM are similar isosceles triangles. Hence

$$\frac{AC}{AK}=\frac{KM}{r} \quad\text{or}\quad AC=\frac{AK\cdot KM}{r}.$$

Hence,

$$\frac{AC}{2r}=\frac{AK\cdot KM}{2r^2}=\frac{2AK\cdot KM}{4r^2}.$$

But

$$\frac{AK}{2r}=\sin\beta \quad\text{and}\quad \frac{KM}{2r}=\cos\beta.$$

Hence $\sin 2\beta = 2\sin\beta\cos\beta$.

(2) $\cos 2\beta = \cos^2\beta - \sin^2\beta$.

Using the same Fig. 1, we get

$$\cos 2\beta = \frac{MC}{2r}=\sqrt{\frac{4r^2-\overline{AC}^2}{2r}}=\sqrt{1-\frac{\overline{AC}^2}{4r^2}}.$$

But

$$AC = (AK\cdot KM)/r.$$

Hence,

$$\cos 2\beta = \sqrt{1-\frac{4\overline{AK}^2\cdot\overline{KM}^2}{16r^4}}.$$

Substituting the values of AK and KM, as previously found, we have

$$\cos 2\beta = \sqrt{1-4\sin^2\beta\cos^2\beta} = \sqrt{(\sin^2\beta+\cos^2\beta)^2-4\sin^2\beta\cos^2\beta} = \cos^2\beta-\sin^2\beta.$$

(3) $\sin 3\beta = 3\sin\beta - 4\sin^3\beta$.

Inscribe in a circle any triangle ABC with the angle at B equal to 3β (Fig. 2). Draw AM, the diameter of the circle. Arc AH = arc HK = arc KC. Draw the chords AK, HC, KC, HM, KM, BH, and BK.

Hence,

$$\sin 3\beta = \frac{AC}{2r}.$$

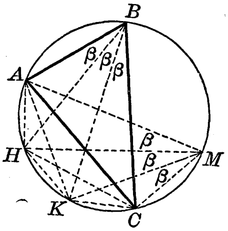

FIG. 2.

$AHKC$ is a concylic quadrilateral. Hence,

$$AC \cdot HK + AH \cdot KC = AK \cdot HC,$$

or

$$AC \cdot AH + \overline{AH}^2 = AK \cdot HC = \overline{AK}^2;$$

whence,

$$AC = (\overline{AK}^2 - \overline{AH}^2)/AH.$$

Hence,

$$\sin 3\beta = (\overline{AK}^2 - \overline{AH}^2)/2r \cdot AH.$$

But

$$AK = HC = 2r \sin 2\beta = 4r \sin\beta \cos\beta,$$

and

$$AH = 2r \sin\beta.$$

Hence,

$$\sin 3\beta = \frac{16r^2 \sin^2\beta \cos^2\beta - 4r^2 \sin^2\beta}{4r^2 \sin\beta} = 3\sin\beta - 4\sin^3\beta.$$

(4) $\cos 3\beta = 4\cos^3\beta - 3\cos\beta$.

From figure 2, we have

$$\cos 3\beta = \frac{MC}{2r}.$$

Also, since $MC^2 = AM^2 - AC^2$,

$$\frac{\overline{MC}^2}{4r^2} = \frac{4r^2 - \overline{AC}^2}{4r^2}.$$

But

$$\overline{AC}^2 = \frac{\overline{AK}^4 - 2\overline{AK}^2 \cdot \overline{AH}^2 + \overline{AH}^4}{\overline{AH}^2}.$$

Hence,

$$\frac{\overline{MC}^2}{4r^2} = 1 - \frac{\overline{AK}^4}{4r^2\overline{AH}^2} + 2\frac{\overline{AK}^2}{4r^2} - \frac{\overline{AH}^2}{4r^2}.$$

Substituting the values of AK and AH, we have

$$\frac{\overline{MC}^2}{4r^2} = 1 - 16\sin^2\beta\cos^4\beta + 8\sin^2\beta\cos^2\beta - \sin^2\beta = 16\cos^6\beta - 24\cos^4\beta + 9\cos^2\beta,$$

or

$$\frac{MC}{2r} = 4\cos^3\beta - 3\cos\beta.$$

Hence,

$$\cos 3\beta = 4\cos^3\beta - 3\cos\beta.$$

Also solved by A. M. HARDING and J. VINCENT BALCH.

CALCULUS.

366. Proposed by I. A. BARNETT, University of Chicago.

Compute the definite integral $\int_a^b \sin^{-1} x dx$, where $0 \leqq a \leqq 1$, $0 \leqq b \leqq 1$, by direct summation.